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On the equivalence between noncommutative and ordinary gauge theories

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Abstract: Recently Seiberg and Witten have proposed that noncommutative gauge theories realized as effective theories on D-brane are equivalent to some ordinary gauge theories. This proposal has been proved, however, only for the Dirac-Born-Infeld action in the approximation of neglecting all derivative terms. In this paper we explicitly construct general forms of the $2n$-derivative terms which satisfy this equivalence under their assumption in the approximation of neglecting $(2n+2)$-derivative terms. We also prove that the D-brane action computed in the superstring theory is consistent with the equivalence neglecting the fourth and higher order derivative terms.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Gauge Symmetry.

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Contents

1. Introduction

Recently it has been realized that the noncommutative geometry has played a profound role in a specific compactification of the Matrix theory [1] and also in superstring theory via D-branes with constant B fields [2]-[6]. From deeper investigation of the noncommutative gauge theory via D-branes, Seiberg and Witten have proposed that the noncommutative gauge theories realized as effective theories on D-branes are equivalent to some ordinary gauge theories [7]. In a single D-brane case, it has been known that the effective action on the brane is Dirac-Born-Infeld action if all derivative terms are neglected [8]-[10]. Thus the Dirac-Born-Infeld action should be consistent with the equivalence in this approximation. Indeed this has been shown in [7].

It is a very natural question whether the equivalence indeed holds beyond the approximation of neglecting all derivative terms, or not. If it holds without the approximation, the forms of derivative corrections have to be highly restricted. Moreover when this equivalence is strong enough, we can determine the effective action completely from it with the help of other requirements and study the dynamics of the D-branes using the action.

In this paper, we show that in the approximation of neglecting the fourth and higher order derivative terms the D-brane action computed in the superstring theory is consistent with the equivalence. Although, this is the Dirac-Born-Infeld action without two-derivative corrections, to show the equivalence we should take into account the orderings of the noncommutative field strength in the Dirac-Born-Infeld action. By taking appropriate ordering it becomes consistent and this is regarded as a non-trivial test of the equivalence.

With the mapping of the ordinary gauge field to noncommutative gauge field given in [7], we also explicitly construct general forms of the two-derivative corrections which satisfy the equivalence relation in the approximation of neglecting the four-derivative terms. Furthermore, we can construct the $2n$-derivative corrections which are consistent with the equivalence in the approximation of neglecting the $(2n+2)$-derivative terms. It should be emphasized that the results obtained in this paper are valid for arbitrary order of the field strength.[1]

On the other hand, in [11] it has been shown that for the bosonic string case the known two-derivative correction [12, 13] is not consistent with the equivalence.[2] This problem can be resolved by considering B-dependent field redefinition of the U(1) gauge field [14]. Therefore in order to constrain the effective action for the bosonic string case, we should include the B-dependent field redefinition. However two-derivative corrections allowed by the equivalence without the B-dependent field redefinition may also be allowed by the equivalence with it [14]. Furthermore, the higher derivative corrections may capture some general structures of the effective action of the D-brane. Therefore the derivative corrections obtained in this paper are probably important.

This paper is organized as follows. In section 2, we briefly review the equivalence between noncommutative and ordinary gauge theories shown in [7]. In section 3 it is shown that the certain noncommutative version of the DBI actions without two-derivative corrections are consistent with the equivalence in the approximation of neglecting the four-derivative terms. We also construct the consistent two-derivative corrections in this approximation. In section 4 we argue that the two-derivative corrections obtained in section 3 exhaust the consistent two-derivative corrections and also generalize these to the $2n$-derivative corrections. Finally section 5 is devoted to conclusion.

2. Noncommutative gauge theory

In this section we briefly review the equivalence between noncommutative and ordinary gauge theories shown in [7]. We consider open strings in flat space, with metric g_{ij}, in the presence of a constant B_{ij} and with a Dp-brane. Here we assume that B_{ij} has rank $p+1$ and $B_{ij} \neq 0$ only for $i, j = 1, \ldots, p+1$. The world-sheet action is

$$S = \frac{1}{4\pi\alpha'} \int_{\Sigma} g_{ij} \partial_a x^i \partial^a x^j - \frac{i}{2} \int_{\partial\Sigma} B_{ij} x^i \partial_\tau x^j - i \int_{\partial\Sigma} A_i(x) \partial_\tau x^i \,, \tag{2.1}$$

where Σ is the string world-sheet, ∂_τ is the tangential derivative along the world-sheet boundary $\partial\Sigma$ and A_i is a background gauge field. In the case that Σ is the

[1] In this paper, we regard $F_{ij} \sim \mathcal{O}(\partial^0)$ and $A_i \sim \mathcal{O}(\partial^{-1})$

[2] In [11] the consistent two-derivative corrections up to the quartic order of the field strength have been considered. As we will see later, the result obtained in this paper reproduce these corrections .

upper half plane parameterized by $-\infty \le \tau \le \infty$ and $0 \le \sigma \le \infty$, the propagator evaluated at boundary points is [8]-[10]

$$\langle x^i(\tau) x^j(\tau')\rangle = -\alpha'(G^{-1})^{ij}\log(\tau-\tau')^2 + \frac{i}{2}\theta^{ij}\epsilon(\tau-\tau')\,, \tag{2.2}$$

where G and θ are the symmetric and antisymmetric tensors defined by

$$(G^{-1})^{ij} + \frac{1}{2\pi\alpha'}\theta^{ij} = \left(\frac{1}{g+2\pi\alpha' B}\right)^{ij}. \tag{2.3}$$

From considerations of the string S-matrix, the B dependence of the effective action for fixed G can be obtained by replacing ordinary multiplication in the effective action for $B=0$ by the $*$ product defined by the formula

$$f(x) * g(x) = e^{\frac{i}{2}\theta^{ij}\frac{\partial}{\partial\xi^i}\frac{\partial}{\partial\zeta^j}} f(x+\xi)g(x+\zeta)\Big|_{\xi=\zeta=0}. \tag{2.4}$$

It is likely that the gauge transformation also becomes noncommutative. In fact, using the point splitting regularization, S is invariant under noncommutative gauge transformation

$$\hat{\delta}\hat{A}_i = \hat{D}_i\lambda\,, \tag{2.5}$$

where covariant derivative $\hat{D}_i$ is defined as

$$\hat{D}_i E(x) = \partial_i E(x) + i\left(E(x) * \hat{A}_i - \hat{A}_i * E(x)\right). \tag{2.6}$$

Conversely, using Pauli-Villars regularization, S is invariant under ordinary gauge transformation

$$\delta A_i = \partial_i\lambda\,. \tag{2.7}$$

Therefore, the effective lagrangian obtained in this way becomes ordinary gauge theory. Thus this theory and the corresponding noncommutative gauge theory are equivalent under the field redefinition $\hat{A} = \hat{A}(A)$ since the coupling constants in the world-sheet theory are the spacetime fields. Because the two different gauge invariance should satisfy $\hat{A}(A) + \hat{\delta}_{\hat{\lambda}}\hat{A}(A) = \hat{A}(A+\delta_\lambda A)$, the mapping of A to $\hat{A}$ for U(1) case is obtained as a differential equation for θ,

$$\begin{aligned}
\delta\hat{A}_i(\theta) = \delta\theta^{kl}\frac{\partial}{\partial\theta^{kl}}\hat{A}_i(\theta) &= -\frac{1}{4}\delta\theta^{kl}\left[\hat{A}_k * (\partial_l\hat{A}_i + \hat{F}_{li}) + (\partial_l\hat{A}_i + \hat{F}_{li}) * \hat{A}_k\right] \\
\delta\hat{F}_{ij}(\theta) = \delta\theta^{kl}\frac{\partial}{\partial\theta^{kl}}\hat{F}_{ij}(\theta) &= \frac{1}{4}\delta\theta^{kl}\Big[2\hat{F}_{ik} * \hat{F}_{jl} + 2\hat{F}_{jl} * \hat{F}_{ik} - \\
&\quad - \hat{A}_k * \left(\hat{D}_l\hat{F}_{ij} + \partial_l\hat{F}_{ij}\right) - \left(\hat{D}_l\hat{F}_{ij} + \partial_l\hat{F}_{ij}\right) * \hat{A}_k\Big]\,,
\end{aligned} \tag{2.8}$$

where

$$\hat{F}_{ij} = \partial_i\hat{A}_j - \partial_j\hat{A}_i - i\hat{A}_i * \hat{A}_j + i\hat{A}_j * \hat{A}_i\,. \tag{2.9}$$

In [15] this map has been derived in a path integral form from D-brane world-volume perspective [16, 17].[3]

[3]Although the differential equation has ambiguities [18], these ambiguities have no physical meaning because they correspond to the field redefinition.

In the approximation of neglecting the derivative terms, the effective lagrangian is the Dirac-Born-Infeld lagrangian

$$\mathcal{L}_{DBI} = \frac{1}{g_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\sqrt{\det(g+2\pi\alpha'(B+F))}\,, \tag{2.10}$$

where $F_{ij} = \partial_i A_j - \partial_j A_i$. Here g_s is the closed string coupling and the normalization of the lagrangian is same as the one taken in [7]. Therefore the equivalent noncommutative gauge theory in the approximation has the following lagrangian

$$\hat{\mathcal{L}}_{DBI} = \frac{1}{G_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\sqrt{\det(G+2\pi\alpha'\hat{F})}\,. \tag{2.11}$$

Note that all the multiplication entering the r.h.s. of (2.11) can be regarded as the ordinary multiplication except those in the definition of $\hat{F}$ because of the approximation. From the requirement $\mathcal{L}_{DBI} = \hat{\mathcal{L}}_{DBI}$ for $F = 0$, the overall normalization G_s should be fixed as $G_s = g_s\sqrt{\det(G)/\det(g+2\pi\alpha' B)}$.

Furthermore, in [7] it has been proposed that the effective action can be written for arbitrary values of θ. More precisely for given physical parameters g_s, g_{ij} and B_{ij} and an auxiliary parameter θ, we define G_s, G_{ij} and a two form Φ_{ij} as

$$\begin{aligned}\left(\frac{1}{G+2\pi\alpha'\Phi}\right)^{ij} &= -\frac{1}{2\pi\alpha'}\theta^{ij} + \left(\frac{1}{g+2\pi\alpha' B}\right)^{ij} \\ G_s &= g_s\left(\det\left(-\frac{1}{2\pi\alpha'}\theta + \frac{1}{g+2\pi\alpha' B}\right)\det(g+2\pi\alpha' B)\right)^{-1/2}.\end{aligned} \tag{2.12}$$

Then the effective action $\hat{S}(G_s, G, \Phi, \theta;\, \hat{F})$, in which the multiplication is the θ-dependent $*$ product, is actually θ-independent, i.e. $\hat{S}(G_s, G, \Phi, \theta;\, \hat{F}) = S(g_s, g, B, \theta = 0;\, F)$. The effective action including Φ may be obtained using a regularization which interpolates between Pauli-Villars and point splitting as in [19]. In this paper, we simply assume this proposal.

In the approximation of neglecting the derivative of F, the equation

$$\delta\mathcal{L}_\Phi = \delta\theta^{kl}\frac{\partial\mathcal{L}_\Phi}{\partial\theta^{kl}}\bigg|_{g_s,g,B,A_i\ \mathrm{fixed}} = \text{total derivative} + \mathcal{O}(\partial^2)\,, \tag{2.13}$$

should hold.[4] Here $\mathcal{L}_\Phi$ is the lagrangian defined as

$$\mathcal{L}_\Phi = \frac{1}{G_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\sqrt{\det(G+2\pi\alpha'(\hat{F}+\Phi))}, \tag{2.14}$$

where the multiplication is the $*$ product except in the definition of $\hat{F}$. Below for simplicity we set $2\pi\alpha' = 1$. The variation of G_s, G and Φ are

$$\begin{aligned}\delta G_s &= \frac{1}{2}G_s\,\mathrm{Tr}(\Phi\delta\theta) \\ \delta G &= G\delta\theta\Phi + \Phi\delta\theta G \\ \delta\Phi &= \Phi\delta\theta\Phi + G\delta\theta G\,,\end{aligned} \tag{2.15}$$

[4]Hereafter δ always denotes $\delta\theta^{kl}\frac{\partial}{\partial\theta^{kl}}$.

and the variation of $\hat{F}$ is

$$\begin{aligned}\delta\hat{F}_{ij} &= -(\hat{F}\delta\theta\hat{F})_{ij} - \hat{A}_k\delta\theta^{kl}\frac{1}{2}(\partial_l + \hat{D}_l)\hat{F}_{ij} + \mathcal{O}(\partial^4) \\ &= -(\hat{F}\delta\theta\hat{F})_{ij} - \hat{A}_k\delta\theta^{kl}\left(\partial_l - \frac{1}{2}\theta^{mn}\partial_n\hat{A}_l\partial_m\right)\hat{F}_{ij} + \mathcal{O}(\partial^4)\,. \end{aligned}\tag{2.16}$$

Following [7], we get

$$\begin{aligned}\delta\left(\frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\right) &= -\frac{1}{2}\frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\times \\ &\times\left(\mathrm{Tr}(\hat{F}\delta\theta) + \left(\frac{1}{G+\hat{F}+\Phi}\right)_{ji}\hat{A}_k\delta\theta^{kl}\frac{1}{2}(\partial_l+\hat{D}_l)\hat{F}_{ij}\right),\end{aligned}\tag{2.17}$$

where the multiplication is the ordinary one except in $\hat{F}$ and $\hat{D}_l$. Now, using

$$\frac{1}{2}(\partial_l + \hat{D}_l)\hat{A}_k - \frac{1}{2}(\partial_k + \hat{D}_k)\hat{A}_l = \hat{D}_l\hat{A}_k - \partial_k\hat{A}_l = \hat{F}_{lk}\,,\tag{2.18}$$

we see that

$$\begin{aligned}&\delta\theta^{kl}(\partial_l + \hat{D}_l)\left(\hat{A}_k\det(G+\hat{F}+\Phi)^{1/2}\right) = \\ &= \delta\theta^{kl}\det(G+\hat{F}+\Phi)^{1/2}\left(\hat{F}_{lk} + \frac{1}{2}\left(\frac{1}{G+\hat{F}+\Phi}\right)_{ji}\hat{A}_k(\partial_l+\hat{D}_l)\hat{F}_{ij}\right) + \mathcal{O}(\partial^4)\,,\end{aligned}\tag{2.19}$$

is a total derivative. Thus we obtain the desired result

$$\delta\left(\frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\right) = \text{total derivative} + \mathcal{O}(\partial^4).\tag{2.20}$$

Note that the computation above shows that the Dirac-Born-Infeld lagrangian (2.14) is θ-independent even in the approximation of neglecting $\mathcal{O}(\partial^4)$ terms.

3. Two-derivative terms

In this section, we will see that certain noncommutative Dirac-Born-Infeld actions are consistent with the equivalence in the approximation of neglecting four-derivative terms. We also give certain two-derivative corrections consistent with the equivalence in the approximation of neglecting $\mathcal{O}(\partial^4)$ terms.

Because the multiplication in the Dirac-Born-Infeld lagrangian $\mathcal{L}_\Phi$ should be replaced by the $*$ product in the approximation, we first consider the ordering of the $\hat{F}_{ij}$ in the Dirac-Born-Infeld lagrangian in which the multiplication is the $*$ product. This lagrangian is relevant in the approximation $\mathcal{O}(\partial^4)$ and denoted as $\hat{\mathcal{L}}_\Phi$.

It seems that there are two natural ways of ordering. The first one is symmetrization of the $(\hat{F}+\Phi)_{ij}$ in $\hat{\mathcal{L}}_\Phi$, as in the non-abelian Dirac-Born-Infeld lagrangian considered in [21]. The another one is as follows. First we expand the square root of determinant using $U_{2n} \equiv \mathrm{Tr}(G^{-1}(\hat{F}+\Phi))^{2n}$. Next keeping the order of $(\hat{F}+\Phi)_{ij}$

in U_{2n} indicated by the symbol Tr, we symmetrize the polynomials of U_{2n}'s. Then replacing all the multiplication by $*$ product, we obtain the $\hat{\mathcal{L}}_\Phi$ from the $\mathcal{L}_\Phi$.

To take the either one of these, we will show

$$\hat{\mathcal{L}}_\Phi = \mathcal{L}_\Phi + \mathcal{O}(\partial^4)\,. \tag{3.1}$$

To show this, we first remember the fact $f * g + g * f = 2fg + \mathcal{O}(\partial^4)$. Thus taking the first way of ordering, we easily see that (3.1) is satisfied since $(\hat{F}+\Phi)_{ij}$ is symmetrized. If we take the second way, using

$$\begin{aligned}
&\mathrm{Tr}\left[(G^{-1}(\hat{F}+\Phi)) * \cdots * (G^{-1}(\hat{F}+\Phi))\right] = \\
&= \mathrm{Tr}\left[(G^{-1}(\hat{F}+\Phi))^{2n}\right] + i\sum_{m=0}^{n-2}(m+1)\theta^{kl}\mathrm{Tr}\left[(G^{-1}\partial_k\hat{F})(G^{-1}(\hat{F}+\Phi))^{n-m-2}\times\right. \\
&\qquad\qquad \left.\times\,(G^{-1}\partial_l\hat{F})(G^{-1}(\hat{F}+\Phi))^{m}\right] + \mathcal{O}(\partial^4) \\
&= \mathrm{Tr}\left[(G^{-1}(\hat{F}+\Phi))^{2n}\right] + \mathcal{O}(\partial^4)\,,
\end{aligned} \tag{3.2}$$

we can also show (3.1). Note that if we take the other way of ordering, (3.1) is not necessarily satisfied.

From (2.20) and (3.1), we conclude that the noncommutative Dirac-Born-Infeld lagrangian with one of these orderings, $\hat{\mathcal{L}}_\Phi$, satisfies the desired equation

$$\delta\hat{\mathcal{L}}_\Phi = \text{total derivative} + \mathcal{O}(\partial^4)\,. \tag{3.3}$$

Therefore this lagrangian without two derivative corrections is allowed by the equivalence in the approximation of neglecting $\mathcal{O}(\partial^4)$, but keeping an arbitrary order of $\hat{F}$. This result is consistent with the calculations of the effective action for the superstring case [12] because in this case there are no two-derivative terms in the effective action.

For the bosonic case, it has been known [11] that the known two-derivative corrections derived from the string four-point amplitude [12] and the β function in the open string σ model [13] is not consistent with the equivalence with (2.9). However it can be shown [14] that if the mapping of A to $\hat{A}$ (2.9) is modified by some field redefinition containing θ, F and Φ, the equivalence is consistent with the result in [12] and [13].

Although this modification should be applied for the bosonic case, we will consider the two-derivative corrections which are consistent with the equivalence using (2.9) in the rest of this section. This is because these corrections can be added consistently in the approximation even if we modify (2.9) and may capture some general structures of the effective action of the D-brane. We will also consider the $2m$-derivative corrections which are consistent with the equivalence using (2.9) in the approximation of neglecting $(2m+2)$-derivative terms in the next section. For the superstring case, it is possible that (2.9) is valid even if we do not neglect the higher-derivative terms. Hence it is important that the determination of these corrections.

Then we will show below that the two-derivative term

$$\hat{\mathcal{L}}_2 = \frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\,L_2\,, \tag{3.4}$$

satisfies

$$\delta\hat{\mathcal{L}}_2 = \text{total derivative} + \mathcal{O}(\partial^4), \tag{3.5}$$

where

$$\begin{aligned}
L_2 &= \left\{a_1\,(h_S)^{mn}(h_S)^{iq}(h_S)^{jp} + a_2\,(h_S)^{mi}(h_S)^{nq}(h_S)^{jp}\right\}\hat{D}_m\hat{F}_{ij}\hat{D}_n\hat{F}_{pq},\\
(h_S)^{ij} &= \left(\frac{1}{G+\hat{F}+\Phi}\right)^{ij}_{\text{sym}} = \frac{1}{2}\left(\frac{1}{G+\hat{F}+\Phi}\right)^{ij} + \frac{1}{2}\left(\frac{1}{G-\hat{F}-\Phi}\right)^{ij}\\
&= \left(\frac{1}{G+\hat{F}+\Phi}G\frac{1}{G-\hat{F}-\Phi}\right)^{ij}, \qquad (3.6)
\end{aligned}$$

and a_1 and a_2 are some constants. Here it is not required to consider the ordering problem of (3.4) because the condition (3.5) means that the term is consistent with the equivalence in the approximation of neglecting $\mathcal{O}(\partial^4)$. How these terms are derived is explained in the next section.

If we define a differential operator

$$\tilde{\delta} = \frac{1}{2}\delta\theta^{kl}\hat{A}_k(\partial_l+\hat{D}_l)\,, \tag{3.7}$$

the differential of $\hat{\mathcal{L}}_2$ is written as

$$\delta\hat{\mathcal{L}}_2 = -\frac{1}{2}\delta\theta^{kl}(\partial_l+\hat{D}_l)\left(\hat{A}_k\hat{\mathcal{L}}_2\right) + \frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\,(\delta+\tilde{\delta})\,L_2\,, \tag{3.8}$$

where the first term of the r.h.s. of (3.8) is a total derivative. Hence we consider the variations of h_S and $\hat{D}\hat{F}$ under $\delta+\tilde{\delta}$.

From

$$(\delta+\tilde{\delta})\hat{F}_{ij} = -(\hat{F}\delta\theta\hat{F})_{ij} + \mathcal{O}(\partial^4)\,, \tag{3.9}$$

it obeys

$$\begin{aligned}
(\delta+\tilde{\delta})h_S{}^{ij} &= -\left(\frac{1}{G+\hat{F}+\Phi}\left((G+\Phi)\delta\theta(G+\Phi) + (\delta+\tilde{\delta})\hat{F}\right)\frac{1}{G+\hat{F}+\Phi}\right)^{ij}_{\text{sym}}\\
&= -\left(\delta\theta - \frac{1}{G+\hat{F}+\Phi}\hat{F}\delta\theta - \delta\theta\hat{F}\frac{1}{G+\hat{F}+\Phi}\right)^{ij}_{\text{sym}} + \mathcal{O}(\partial^4)\\
&= \left(h_S(\hat{F}\delta\theta) + (\delta\theta\hat{F})h_S\right)^{ij} + \mathcal{O}(\partial^4)\,. \qquad (3.10)
\end{aligned}$$

Remembering that $[\delta,\partial_m]=0$ and that $\hat{D}$ explicitly depends on θ through $*$ product, we see that the commutation relation between $\hat{D}$ and δ is

$$\begin{aligned}
[\delta,\hat{D}_m]E &= [\delta,\partial_m]E + i[E,\delta\hat{A}_m] + \delta\theta^{kl}\partial_k\hat{A}_m\partial_l E + \mathcal{O}(\partial^5 E)\\
&= i\left[E, -\frac{1}{2}\delta\theta^{kl}\hat{A}_k(\partial_l\hat{A}_m + \hat{F}_{lm})\right] + \delta\theta^{kl}\partial_k\hat{A}_m\partial_l E + \mathcal{O}(\partial^5 E)\,, \qquad (3.11)
\end{aligned}$$

where E is an arbitrary function. The computation of the commutation relation between $\hat{D}$ and $\tilde{\delta}$ can be carried out straightforwardly

$$
\begin{aligned}
[\tilde{\delta}, \hat{D}_m]E &= \tilde{\delta}\left(\partial_m E + i[E, \hat{A}_m]\right) - \hat{D}_m\left(\frac{1}{2}\hat{A}_k\delta\theta^{kl}(\partial_l + \hat{D}_l)E\right) \\
&= -\frac{1}{2}\delta\theta^{kl}\left(\hat{D}_m\hat{A}_k(\partial_l + \hat{D}_l)E + i\hat{A}_k[E, \hat{F}_{ml} - \partial_l\hat{A}_m]\right) + \mathcal{O}(\partial^5 E)\,. \quad (3.12)
\end{aligned}
$$

After some calculations using (3.11) and (3.12), we can find a simple result

$$
[\delta + \tilde{\delta}, \hat{D}_m]E = -(\hat{F}\delta\theta)_m{}^l\,(\hat{D}_l E) + \mathcal{O}(\partial^5 E)\,, \quad (3.13)
$$

and then we obtain

$$
(\delta + \tilde{\delta})\hat{D}_m\hat{F}_{ij} = -\left((\hat{D}_m\hat{F})\delta\theta\hat{F}\right)_{ij} - \left(\hat{F}\delta\theta(\hat{D}_m\hat{F})\right)_{ij} - (\hat{F}\delta\theta)_m{}^l\,(\hat{D}_l\hat{F}_{ij}) + \mathcal{O}(\partial^5). \quad (3.14)
$$

This and (3.10) imply that $(\delta + \tilde{\delta})L_2 = 0$. Thus we conclude that $\delta\hat{\mathcal{L}}_2 =$ total derivative $+\,\mathcal{O}(\partial^6)$. However $\mathcal{O}(\partial^4)$ terms may exist if the ordinary multiplication in (3.4) is replaced by $*$ product. Thus only (3.5) is meaningful because we have not considered the ordering problems of (3.4).

Now we discuss the expansion about F of two-derivative corrections of the effective lagrangian (3.4) with $B = \Phi = \theta = 0$ and $g_{ij} = \delta_{ij}$. In this commutative description, the Dirac-Born-Infeld lagrangian (2.11) becomes $1/(g_s(2\pi)^{\frac{p+1}{2}})\sqrt{\det(1+F)}$. Using $\det(G + \hat{F} + \Phi)^{1/2} = 1 - \frac{1}{4}\operatorname{Tr}F^2 + \mathcal{O}(F^4)$ and $h_S = \frac{1}{1-F^2} = 1 + F^2 + \mathcal{O}(F^4)$, we see

$$
\begin{aligned}
\hat{\mathcal{L}}_2 = a_1\frac{1}{g_s}\Bigg[&\left(1 - \frac{1}{4}\operatorname{Tr}F^2\right)\partial_m F_{ij}\partial_m F_{ji} + \\
&+ (F^2)_{mn}\partial_m F_{ij}\partial_m F_{ji} + 2(F^2)_{ik}\partial_m F_{ij}\partial_m F_{jk}\Bigg] + \\
+ a_2\frac{1}{g_s}\Bigg[&\left(1 - \frac{1}{4}\operatorname{Tr}F^2\right)\partial_m F_{im}\partial_n F_{ni} + \\
&+ 2(F^2)_{mj}\partial_m F_{ij}\partial_n F_{ni} + (F^2)_{iq}\partial_m F_{im}\partial_n F_{nq}\Bigg] + \mathcal{O}(F^4\partial F\partial F) \quad (3.15)
\end{aligned}
$$

Here following [12], we define a basis of terms of order $F^2\partial F\partial F$ as

$$
\begin{aligned}
J_1 &= F_{kl}F_{lk}\partial_n F_{ij}\partial_n F_{ji} & J_2 &= F_{kl}F_{li}\partial_n F_{ij}\partial_n F_{jk} \\
J_3 &= F_{ni}F_{im}\partial_n F_{kl}\partial_m F_{lk} & J_4 &= F_{kl}F_{lk}\partial_n F_{ni}\partial_m F_{im} \\
J_5 &= -F_{jk}F_{km}\partial_n F_{ni}\partial_m F_{ij} & J_6 &= F_{kl}F_{lk}\partial_m\partial_m F_{ij}F_{ji} \\
J_7 &= \partial_m\partial_m F_{ij}F_{jk}F_{kl}F_{li}\,. & & \quad (3.16)
\end{aligned}
$$

Therefore after some computations, we obtain

$$
\begin{aligned}
\hat{\mathcal{L}}_2 &= a_1\frac{1}{g_s}\left(\partial_m F_{ij}\partial_m F_{ji} - \frac{1}{4}J_1 + 2J_2 + J_3\right) + \\
&\quad + a_2\frac{1}{g_s}\left(\frac{1}{2}\partial_m F_{ij}\partial_m F_{ji} - J_5 + \frac{1}{8}J_6 - \frac{1}{2}J_7\right) + \cdots, \quad (3.17)
\end{aligned}
$$

where the ellipsis represent total derivative terms and $\mathcal{O}(F^4\partial F\partial F)$ terms. This is same as the effective lagrangian obtained in [11]. Thus the result obtained in this paper implies that the one obtained in [11] is consistent even in the arbitrary order of F. It is noted that in [11] only the equivalence without assuming the existence of Φ is required.

4. General forms of the derivative corrections

In this section, we discuss the other two-derivative corrections which satisfy (3.5) and also the higher derivative corrections.

Requiring the noncommutative gauge invariance and the gauge invariance for the B field, the most general two derivative terms are

$$\frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\Big[T^{ijklmn}(G^{-1},\hat{F}+\Phi)\,\hat{D}_m\hat{F}_{ij}\hat{D}_n\hat{F}_{kl}+ + T^{ijmn}(G^{-1},\hat{F}+\Phi)\hat{D}_m\hat{D}_n\hat{F}_{ij}\Big]\,, \tag{4.1}$$

where T^{ijklmn} and T^{ijmn} are arbitrary tensors constructed from $(G^{-1})^{pq}$, $M_{pq}\equiv(\hat{F}+\Phi)_{pq}$, $\det G$ and $\det M$. The tensors T should not depend on $(M^{-1})^{pq}$ because it has a singularity at $M=0$.

We can easily generalize the invariance problem under δ of two-derivative terms to the one of higher derivative terms. Thus below we will look for the lagrangian $\hat{\mathcal{L}}_m$ with m-derivative which satisfies the invariant condition

$$\delta\hat{\mathcal{L}}_m = \text{total derivative} + \mathcal{O}(\partial^{m+2})\,, \tag{4.2}$$

which is the condition consistent with the equivalence in the approximation of neglecting $\mathcal{O}(\partial^{m+2})$. To do this, let us consider a $(2n,0)$ tensor $T^{p_1p_2\cdots p_{2n}}$ constructed from G^{-1} and M. It can be written as

$$T^{p_1p_2\cdots p_{2n}} = \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty} C_{\{k\}}\,(M^{k_1})^{p_1p_2}(M^{k_2})^{p_3p_4}\cdots(M^{k_n})^{p_{2n-1}p_{2n}}\,, \tag{4.3}$$

where $C_{\{k\}}$ is some function of the scalars constructed from M and G and $(M^{k_i})^{p_{2i-1}p_{2i}}$ means $((G^{-1}M)^{k_i}G^{-1})^{p_{2i-1}p_{2i}}$. We also consider a $(0,2n)$ tensor $\hat{J}_{p_1p_2\cdots p_{2n}}$ such that

$$\hat{J}_{p_1p_2\cdots p_{2n}} = \Big\{(\hat{D}\cdots\hat{D}\hat{F})\cdots(\hat{D}\cdots\hat{D}\hat{F})\Big\}_{p_1p_2\cdots p_{2n}}\,. \tag{4.4}$$

For given n and m, where m is the number of derivative $\hat{D}$ in $\hat{J}$, there are finite number s of independent $\hat{J}_{p_1p_2\cdots p_{2n}}$ under the identification using Bianchi identity. We note that the total divergence terms should not be used for the identification. Taking a basis of these $\hat{J}^{(i)}$, where $1\le i\le s$, we will study the invariance of

$$\hat{\mathcal{L}}_m = \frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\sum_{i=1}^{s}\sum_{p_1,\cdots,p_{2n}}(T_{(i)})^{p_1p_2\cdots p_{2n}}\,(\hat{J}^{(i)})_{p_1p_2\cdots p_{2n}}\,, \tag{4.5}$$

where $(T_{(i)})$ is the tensor of the form (4.3) with the coefficients $C^{(i)}_{\{k\}}$. As like the derivation of (3.8), we can show that

$$\delta\hat{\mathcal{L}}_m = \text{total derivative} + \frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\,(\delta+\tilde{\delta})\times$$
$$\times\left(\sum_{i=1}^{s}\sum_{p_1,\cdots,p_{2n}}(T_{(i)})^{p_1\cdots p_{2n}}\,(\hat{J}^{(i)})_{p_1\cdots p_{2n}}\right). \qquad (4.6)$$

However there is a possibility of cancellation between the variations under δ of the terms with different n's. Note that this cancellation can occur only between the variations of the terms with n and $n+1$. We will consider this later.

In order to proceed further, we require the invariance of $\hat{\mathcal{L}}_m$ with $\hat{F}=0$ first. From (4.6), we have conditions for the invariance as

$$\sum_{i=1}^{s}\sum_{p_1,\cdots,p_{2n}}(\delta T_{(i)})^{p_1p_2\cdots p_{2n}}\,(\hat{J}^{(i)})_{p_1\cdots p_{2n}} = 0\,. \qquad (4.7)$$

It can be shown that $\delta G^{-1}|_{G=1} = -\left(\Phi\delta\theta+\delta\theta\Phi\right)$ and

$$\delta\left((G^{-1}\Phi)^kG^{-1}\right)\Big|_{G=1} = -\left(\delta\theta\Phi^{k+1}+\Phi\delta\theta\Phi^{k}+\cdots+\Phi^{k+1}\delta\theta\right)+$$
$$+\left(\delta\theta\Phi^{k-1}+\Phi\delta\theta\Phi^{k-2}+\cdots+\Phi^{k-1}\delta\theta\right), \qquad (4.8)$$

where $k\geq 1$. Here we have set $G=1$ after operating δ for notational simplicity. Therefore to satisfy (4.7), we have to take

$$(T_{(i)})^{p_1\cdots p_{2n}} = C^{(i)}\,(h_S)^{p_1p_2}(h_S)^{p_3p_4}\cdots(h_S)^{p_{2n-1}p_{2n}}\,, \qquad (4.9)$$

where $C^{(i)}$ is some function of the scalars constructed from M and G. We also see that this $C^{(i)}$ should be some constant since $\delta\,\mathrm{Tr}((G^{-1}\Phi)^{2k})|_{G=1} = 2k\mathrm{Tr}(\delta\theta(\Phi^{2k-1}-\Phi^{2k+1}))$.

Now we require the condition (4.7) without taking $\hat{F}=0$. Using (3.10) and (3.14) as in the previous section, one can easily show that the condition (4.7) is satisfied for

$$\hat{\mathcal{L}}_m = \frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\sum_{i=1}^{s_1}C^{(i)}\sum_{p_1,\cdots,p_{2n}}(h_S)^{p_1p_2}(h_S)^{p_3p_4}\cdots(h_S)^{p_{2n-1}p_{2n}}\,\bar{J}^{(i)}_{p_1p_2\cdots p_{2n}}\,, \qquad (4.10)$$

where $\{\bar{J}^{(i)}_{p_1p_2\cdots p_{2n}}\}$ is a basis of the form

$$((\hat{D}\hat{F})\cdots(\hat{D}\hat{F}))_{p_1p_2\cdots p_{2n}}\,. \qquad (4.11)$$

Hence this term is allowed by the equivalence in the approximation. In particular, for two-derivative terms, (4.10) is equivalent to (3.4).

The terms containing $(\hat{D})^N\hat{F}$ with $N \geq 2$ are other candidates, but they can not satisfy the condition (4.7) generically because there are contributions from $(\hat{D})^N\hat{F}$ which can not be canceled by the ones from h_S for the variation of $\hat{\mathcal{L}}_m$ under δ as seen from (3.13) with $E = (\hat{D})^{N-1}\hat{F}$. However, in some cases, these are absent because of the symmetry for the indices. For example, we consider

$$\hat{\mathcal{L}}_2 \sim (h_S)^{ip}(h_S)^{jq}[\hat{D}_p, \hat{D}_q]\hat{F}_{ij} \,. \tag{4.12}$$

Remember that

$$\begin{aligned}(\delta+\tilde{\delta})(\hat{D}_p\hat{D}_q\hat{F}_{ij}) = -(\delta\theta)^{lk}\Big((\hat{D}_p\hat{F}_{ql})(\hat{D}_k\hat{F}_{ij}) + (\hat{D}_q\hat{F}_{il})(\hat{D}_p\hat{F}_{kj})& \\ + (\hat{D}_p\hat{F}_{il})(\hat{D}_q\hat{F}_{kj})\Big) + \cdots ,&\end{aligned} \tag{4.13}$$

where the ellipsis represents the terms which are canceled by the contribution from h_S and $\mathcal{O}(\partial^6)$. Thus we obtain $(\delta+\tilde{\delta})((h_S)^{ip}(h_S)^{jq}[\hat{D}_p, \hat{D}_q]\hat{F}_{ij}) = \mathcal{O}(\partial^6)$ from the Bianchi identity and the symmetries of the indices. Therefore (4.12) is also allowed by the equivalence in the approximation though this vanishes at $\theta = 0$. Note that (4.12) is the only allowed term with two-derivative and one $\hat{F}$ because of the Bianchi identity and the symmetry for the indices of $\hat{F}_{ij}$.

There are invariant combinations of the terms with different numbers of indices which implies different numbers of $\hat{F}$. To illustrate these, we consider

$$\hat{\mathcal{L}}_4^A = \frac{C}{G_s}\det(G+\hat{F}+\Phi)^{1/2}(h_S)^{pq}(h_S)^{tu}(h_S)^{ic}(h_S)^{jd}(\hat{D}_p\hat{D}_q\hat{F}_{ij})(\hat{D}_t\hat{D}_u\hat{F}_{cd}) \,, \tag{4.14}$$

where C is some constant. In this case, using (4.13) the unnecessary terms are easily computed as

$$\begin{aligned}\delta\hat{\mathcal{L}}_4^A \sim -2\frac{C}{G_s}\det(G+\hat{F}+\Phi)^{1/2}(h_S)^{pq}(h_S)^{tu}(h_S)^{ic}(h_S)^{jd} \times& \\ \times(\delta\theta)^{lk}(\hat{D}_p\hat{F}_{ql})(\hat{D}_k\hat{F}_{ij})(\hat{D}_t\hat{D}_u\hat{F}_{cd}) \,,&\end{aligned} \tag{4.15}$$

where we neglect the terms which are canceled by the contribution from h_S and $\mathcal{O}(\partial^8)$. Let us define

$$(h_A)^{ij} = \left(\frac{1}{G+\hat{F}+\Phi}\right)^{ij}_{\text{anti.sym}} = \left(\frac{1}{G+\hat{F}+\Phi}(\hat{F}+\Phi)\frac{1}{G-\hat{F}-\Phi}\right)^{ij} , \tag{4.16}$$

which obeys

$$\begin{aligned}(\delta+\tilde{\delta})h_A{}^{ij} &= -\left(\frac{1}{G+\hat{F}+\Phi}\Big((G+\Phi)\delta\theta(G+\Phi) + (\delta+\tilde{\delta})\hat{F}\Big)\frac{1}{G+\hat{F}+\Phi}\right)^{ij}_{\text{anti.sym}} \\ &= -(\delta\theta)^{ij} + \Big(h_A(\hat{F}\delta\theta) + (\delta\theta\hat{F})h_A\Big)^{ij} + \mathcal{O}(\partial^4) \,.\end{aligned} \tag{4.17}$$

Then it can be seen that

$$\delta(\hat{\mathcal{L}}_4^A + \hat{\mathcal{L}}_4^B + \hat{\mathcal{L}}_4^C) \sim 0\,, \tag{4.18}$$

where

$$\begin{aligned} \hat{\mathcal{L}}_4^B &= -2\frac{C}{G_s}\det(G+\hat{F}+\Phi)^{1/2}(h_S)^{pq}(h_S)^{tu}(h_S)^{ic}(h_S)^{jd}\times \\ &\quad \times(h_A)^{lk}(\hat{D}_p\hat{F}_{ql})(\hat{D}_k\hat{F}_{ij})(\hat{D}_t\hat{D}_u\hat{F}_{cd})\,, \\ \hat{\mathcal{L}}_4^C &= \frac{C}{G_s}\det(G+\hat{F}+\Phi)^{1/2}(h_S)^{pq}(h_S)^{tu}(h_S)^{ic}(h_S)^{jd}(h_A)^{lk}(h_A)^{ef}\times \\ &\quad \times(\hat{D}_p\hat{F}_{ql})(\hat{D}_k\hat{F}_{ij})(\hat{D}_t\hat{F}_{ue})(\hat{D}_f\hat{F}_{cd})\,. \end{aligned} \tag{4.19}$$

As this example, for general terms of the form

$$\hat{\mathcal{L}}_m = \frac{1}{G_s}\det(G+\hat{F}+\Phi)^{1/2}\sum_{i=1}^{s_1}C^{(i)}\sum_{p_1,\cdots,p_{2n}}(h_S)^{p_1p_2}(h_S)^{p_3p_4}\cdots(h_S)^{p_{2n-1}p_{2n}}\;\hat{J}^{(i)}_{p_1p_2\cdots p_{2n}}\,, \tag{4.20}$$

we may construct the invariant combinations by adding certain terms like $\hat{\mathcal{L}}_4^B$ and $\hat{\mathcal{L}}_4^C$.

Therefore we conclude that the general forms of the allowed m-derivative corrections in the approximation of neglecting $\mathcal{O}(\partial^{m+2})$ are given by (4.10) and (4.20) with certain terms like $\hat{\mathcal{L}}_4^B$ and $\hat{\mathcal{L}}_4^C$.

Finally we study the behavior of the derivative corrections at $\theta = 0$ in the zero slope limit of [7], $\alpha' \sim \epsilon^{1/2}$, $g_{ij} = \epsilon\delta_{ij}$ with $\epsilon \to 0$. In [7] it has been shown that

$$\mathcal{L}_{DBI} = \frac{1}{g_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\left(|\mathrm{Pf(F+B)}| - \frac{\epsilon^2}{4}|\mathrm{Pf(F+B)}|\,\mathrm{Tr}\,\frac{1}{(F+B)^2} + \mathcal{O}(\epsilon^3)\right), \tag{4.21}$$

where the first term is a constant plus a total derivative. We can show that

$$h_S = -\frac{\epsilon}{(2\pi\alpha')^2}\frac{1}{(F+B)^2} \sim \mathcal{O}(\epsilon^0)\,, \tag{4.22}$$

and

$$h_A = -\frac{1}{2\pi\alpha'}\frac{1}{(F+B)} \sim \mathcal{O}(\epsilon^{-1/2})\,. \tag{4.23}$$

From the dimensional analysis, the constants $C^{(i)}$ and C in (4.10) and (4.20), respectively, are restricted. Indeed, we see that C or $C^{(i)} \sim \alpha'^{-(p+1)/2+n_S+n_A}$, where n_S and n_A are the number of the h_S and h_A in the lagrangians $\hat{\mathcal{L}}$, respectively. Thus

$$\begin{aligned} \hat{\mathcal{L}} &\sim \frac{1}{g_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\times \\ &\quad \times\left(\epsilon^{\frac{n_S}{2}}|\mathrm{Pf(F+B)}|\frac{1}{(F+B)^{2n_S}}\frac{1}{(F+B)^{n_A}}J_{p_1p_2\cdots p_{2(n_S+n_A)}} + \mathcal{O}(\epsilon^{\frac{n_S}{2}+2})\right), \end{aligned} \tag{4.24}$$

where J is $\hat{J}^{(i)}$ or $\tilde{J}$ with $\hat{D} = \partial$ and $\hat{F} = F$. This is negligible compared with $\mathcal{L}_{DBI}$ if $n_s > 4$. Therefore for the superstring case, the only remaining derivative corrections in the limit are the terms like $\hat{\mathcal{L}}_4^A + \hat{\mathcal{L}}_4^B + \hat{\mathcal{L}}_4^C$. This result may have application for a deeper understanding of the relation between the instanton on the noncommutative space [22] and the instanton solution in the Dirac-Born-Infeld lagrangian with nonzero B field [23, 7, 24].

5. Conclusion

We have considered the derivative corrections to the Dirac-Born-Infeld action consistent with the equivalence between the noncommutative gauge theories and the ordinary gauge theory. In particular, we have shown that in the approximation of neglecting the fourth and higher order derivative terms the D-brane action computed in the superstring theory is consistent with the equivalence.

We have also explicitly constructed the the general forms of the $2n$-derivative corrections which satisfy this equivalence relation in the approximation of neglecting the $(2n+2)$-derivative terms. It may capture some general structures of the effective action of the D-brane.

It is interesting to generalize the results obtained in this paper to the effective theories on several D-branes. In this case, we should treat the non-abelian gauge fields, so that the ordering problems exist even for the ordinary gauge fields which have not been solved yet. Thus the constraints using the equivalence are expected to be important for determination of the effective action on the several D-branes.

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Note added. As this article was being completed, we received the preprint [25] which give the derivative corrections for the Dirac-Born-Infeld lagrangian which is invariant under a simplified version [17, 16] of the Seiberg-Witten map. The two-derivative correction obtained in [25]

$$g_{kl} g_{pq} \left(\frac{1}{g+F}\right)^{ij} \partial_i \left(\frac{1}{g+F}\right)^{kp} \partial_i \left(\frac{1}{g+F}\right)^{lq} = -(h_S)^{ij} \mathrm{Tr}\left(h_S(\partial_i F) h_S(\partial_j F)\right), \tag{5.1}$$

coincides with the $\hat{L}_2$ obtained in this paper with $a_1 = -1$ and $a_2 = 0$. Note that on the computation of the expansion about F, some terms are omitted in eq. (4) in [25].

References

[1] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [hep-th/9711162].

[2] M.R. Douglas and C. Hull, *D-branes and the noncommutative torus*, *J. High Energy Phys.* **02** (1998) 008 [hep-th/9711165].

[3] Y.-K.E. Cheung and M. Krogh, *Noncommutative geometry from 0-branes in a background B-field*, *Nucl. Phys.* **B 528** (1998) 185 [hep-th/9803031].

[4] T. Kawano and K. Okuyama, *Matrix theory on noncommutative torus*, *Phys. Lett.* **B 433** (1998) 29 [hep-th/9803044].

[5] C.-S. Chu and P.-M. Ho, *Noncommutative open string and D-brane*, *Nucl. Phys.* **B 550** (1999) 151 [hep-th/9812219]; *Constrained quantization of open string in background b field and noncommutative D-brane*, hep-th/9906192.

[6] V. Schomerus, *D-branes and deformation quantization*, *J. High Energy Phys.* **06** (1999) 030 [hep-th/9903205].

[7] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[8] E.S. Fradkin and A.A. Tseytlin, *Nonlinear electrodynamics from quantized strings*, *Phys. Lett.* **B 163** (1985) 123.

[9] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, *String loop corrections to beta functions*, *Nucl. Phys.* **B 288** (1987) 525.

[10] A. Abouelsaood, C.G. Callan, C.R. Nappi and S.A. Yost, *Open strings in background gauge fields*, *Nucl. Phys.* **B 280** (1987) 599.

[11] Y. Okawa, *Derivative corrections to Dirac-Born-Infeld lagrangian and non-commutative gauge theory*, *Nucl. Phys.* **B 566** (2000) 348 [hep-th/9909132].

[12] O.D. Andreev and A.A. Tseytlin, *Partition function representation for the open superstring effective action: cancellation of mobius infinities and derivative corrections to Born-Infeld lagrangian*, *Nucl. Phys.* **B 311** (1988) 205.

[13] O.D. Andreev and A.A. Tseytlin, *Two loop beta function in the open string sigma model and equivalence with string effective equations of motion*, *Mod. Phys. Lett.* **A 3** (1988) 1349.

[14] Y. Okawa and S. Terashima, *Constraints on effective Lagrangian of D-branes from non-commutative gauge theory,* hep-th/0002194.

[15] K. Okuyama, *A path integral representation of the map between commutative and noncommutative gauge fields*, hep-th/9910138.

[16] N. Ishibashi, *A relation between commutative and noncommutative descriptions of D-branes*, `hep-th/9909176`.

[17] L. Cornalba and R. Schiappa, *Matrix theory star products from the Born-Infeld action*, `hep-th/9907211`;
L. Cornalba, *D-brane physics and noncommutative Yang-Mills theory*, `hep-th/9909081`.

[18] T. Asakawa and I. Kishimoto, *Comments on gauge equivalence in noncommutative geometry*, *J. High Energy Phys.* **11** (1999) 024 [`hep-th/9909139`].

[19] O. Andreev and H. Dorn, *On open string sigma-model and noncommutative gauge fields*, `hep-th/9912070`.

[20] M. Kreuzer and J.-G. Zhou, *Lambda-symmetry and background independence of noncommutative gauge theory on* $\mathbb{R}^n$, *J. High Energy Phys.* **01** (2000) 011 [`hep-th/9912174`].

[21] A.A. Tseytlin, *On non-abelian generalisation of the Born-Infeld action in string theory*, *Nucl. Phys.* **B 501** (1997) 41 [`hep-th/9701125`];
K. Hashimoto, *Generalized supersymetric boundary state*, `hep-th/9909095`.

[22] N. Nekrasov and A. Schwarz, *Instantons on noncommutative* $\mathbb{R}^4$ *and* $(2,0)$ *superconformal six dimensional theory*, *Comm. Math. Phys.* **198** (1998) 689 [`hep-th/9802068`].

[23] S. Terashima, U(1) *instanton in Born-Infeld action and noncommutative gauge theory*, `hep-th/9911245`.

[24] M. Mariño, R. Minasian, G. Moore and A. Strominger, *Nonlinear instantons from supersymmetric p-branes*, *J. High Energy Phys.* **01** (2000) 005 [`hep-th/9911206`].

[25] L. Cornalba, *Corrections to the abelian Born-Infeld action arising from noncommutative geometry*, `hep-th/9912293`.